Eastfield College
Math 1314, College Algebra

Educational Course Objectives: MATH 1314: College Algebra

After completing this course, the student will be able to:
E1. Simplify numerical and algebraic expressions involving order of operations and properties of exponents.
E2. Factor polynomial expressions.
E3. Multiply and divide polynomials.
E4. Solve and graph linear and absolute value equations and inequalities.
E5. Perform basic operations using complex numbers.
E6. Determine the domain and range of functions.
E7. Perform basic operations using complex numbers.
E8. Find the inverse of a function.
E9. Graph polynomial, rational exponential, logarithmic, and special functions.
E10. Simplify exponential and logarithmic expressions.
E11. Solve polynomial, rational radical, exponential, and logarithmic equations.
E12. Solve quadratic inequalities.
E13. Perform basic matrix operations.
E14. Use the Binomial Expansion.
E15. Find a term and the sum of arithmetic and geometric sequences and series.
E16. Solve a system of linear equations algebraically and by using Cramer's Rule.
E17. Use the principles of mathematical induction.

STUDY GUIDE*

E.1.

1. Simplify the algebraic expression $6 - 5[8 - (2y - 4)]$
   Answer: $10y - 54$

2. Simplify the expression $14x^2 + 5 - [7(x^2 - 2) + 4]$
   Answer: $7x^2 + 15$

3. Simplify the exponential expression $(-5x^4y)(-6x^7y^{11})$
   Answer: $30x^{11}y^{12}$

4. Simplify the exponential expression $(\frac{-15a^4b^2}{5a^{10}b^{-3}})^3$
   Answer: $\frac{-27b^{15}}{a^{18}}$

5. Simplify the radical expression $\sqrt[3]{54xy^3} - y\sqrt[3]{128x}$
   Answer: $-y\sqrt[3]{2x}$

E.2.

6. Factor the polynomial $x^3 + 3x^2 + 4x + 2$
   Answer: $(x + 1)(x + 1 + i)(x + 1 - i)$

7. Factor the trinomial, or state the trinomial is prime; $6x^2 - 7xy - 5y^2$
   Answer: $(3x - 5y)(2x + y)$

8. Factor and simplify the algebraic expression by rationalizing the denominator; $12x^{-\frac{3}{4}} + 6x^{\frac{1}{4}}$
   Answer: $\frac{6\sqrt[4]{x}(x+2)}{x}$

9. Factor completely, $(y + 1)^3 + 1$
   Answer: $(y + 2)(y^2 + 3y + 3)$

E.3

10. Multiply the expression $\frac{x^2-4}{x^2-4x+4} \cdot \frac{2x-4}{x+2}$.
    Answer: 2

11. Multiply or divide the expression $\frac{x^2+x-12}{x^2+x-30} \cdot \frac{x^2+5x+6}{x^2-2x-3} \div \frac{x+3}{x^2+7x+6}$.
    Answer: $\frac{(x+4)(x+2)}{(x-5)}$

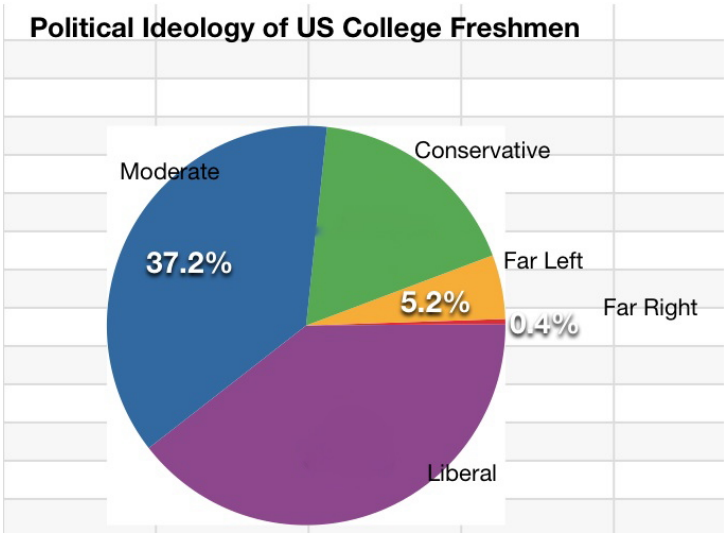
E.4

12. Solve and check the linear equation $2(x-1)+3 = x-3(x+1)$.
Answer: $x = -1$

13. Solve and check the linear equation $5 + \frac{x-2}{3} = \frac{x+3}{8}$.
Answer: $x = -19$

14. The average cost of tuition and fees at public four-year college in the United States can be modeled by the formula $T = 165x + 2771$, where T represents the average cost of tuition and fees for the school year ending x years after 1996. When will tuition and fees at public US colleges average \$5576?
Answer: By the year 2013.

15. The circle graph shows the political ideology of U.S. college freshmen. The percentage of liberals exceeds twice that of conservatives by 4.4%. Find the percentage of liberals and percentage of conservatives.



Answer: Liberal 39.6%, Conservative 17.6%

16. Solve the absolute value equation; $|1+2x| = 5$.
Answer: $x = -3$ or $x = 2$

17. Solve the absolute value equation or indicate that the equation has no solution, $2|4 - \frac{5}{2}x| + 6 = 18$
Answer: $x = 4, x = \frac{-4}{5}$

18.Solve the absolute value inequality. $12 < |-2x + \frac{6}{7}| + \frac{3}{7}$
Answer: $(-\infty, -\frac{75}{14}) \cup (\frac{87}{14}, \infty)$

19. Solve the linear inequality $8x - 11 \leq 3x - 13$.
Answer: $(-\infty, -\frac{2}{5}]$ or

E.5

20. Perform the indicated operations and write the result in standard form $(a + bi)$.

$$\sqrt{-12}(\sqrt{-4} - \sqrt{2})$$

Answer: $-4\sqrt{3} - 2i\sqrt{6}$

21. Divide and express the result in standard form $(a + bi)$.

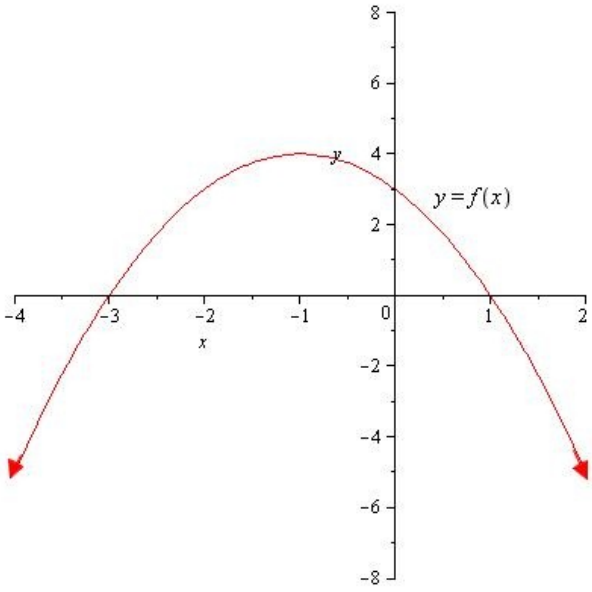
$$\frac{3 - 4i}{4 + 3i}$$

Answer: $-i$

E.6

22. Find the domain of the function $f(x) = \frac{1}{\sqrt{4-x^2}}$. Write your answer in set notation.
Answer: Domain: $\{x| -2 < x < 2\}$

23. Use the graph to determine the function's domain and range, the intervals where the function is increasing and decreasing and the coordinate of the relative maximum.



Answer: Domain: $(-\infty, \infty)$, Range: $(-\infty, 4]$, Increasing: $(-\infty, -1)$, Decreasing: $(-1, \infty)$ and
Relative Maximum: The point $(-1, 4)$.

E.7

24. Let $f(x) = \sqrt{x-3}$ and $g(x) = \sqrt{x+1}$. Find the domain of $f + g$
    Answer: $[3, \infty)$

25. Let $f(x) = x^2 + 4$ and $g(x) = \sqrt{1-x}$. Find $(f \circ g)(x)$.
    Answer: $(fog)(x) = 5 - x$.

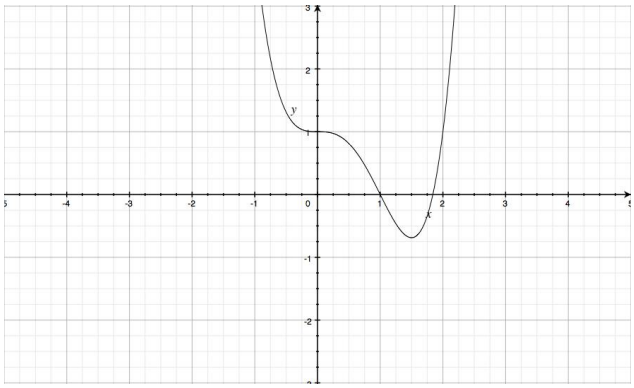
26. Given $f(x) = \frac{4}{x+2}$ and $g(x) = \frac{1}{x}$, find the domain of $f \circ g$.
    Answer: Domain: $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 0) \cup (0, \infty)$

E.8

27. Show that each function is the inverse of the other: $f(x) = 4x - 7$ and $g(x) = \frac{x+7}{4}$.
    Answer: Yes, the functions are inverse of each other. $f(g(x)) = x$ and $g(f(x)) = x$.

28. Find the inverse of the function $f(x) = \sqrt{x-1}$.
    Answer: $f^{-1}(x) = x^2 + 1$ for $x \geq 0$

29. Does the graph below represent a function that has inverse function?



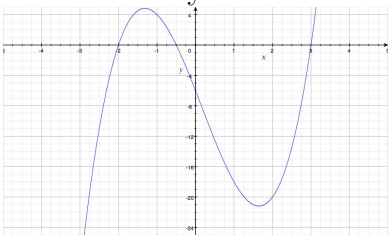
Answer: No, notice that horizontal lines can be drawn and intersect the graph more than once.

30. The formula $y = f(x) = \frac{9}{5}x + 32$ is used to convert from x degrees Celsius to y degrees Fahrenheit. Find the formula to convert from y degrees Fahrenheit to x degrees Celsius. Show that this formula is the inverse function of $f(x)$.
    Answer: $x = \frac{5}{9}(y - 32)$ is the inverse, then $f(\frac{5}{9}(y-32)) = \frac{9}{5}(\frac{5}{9}(y-32)) + 32 = y$, therefore the formula $x = \frac{5}{9}(y - 32)$ is the inverse of $f(x)$.

E.9

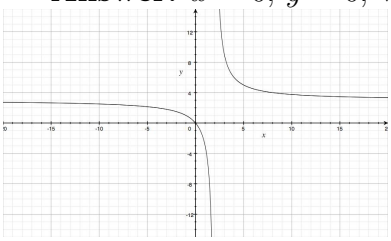
31. Graph the polynomial function: $f(x) = 2x^3 - x^2 - 13x - 6$. Indicate the graph's end behavior, the x-intercepts, state whether the graph crosses the x-axis or touches the x-axis, indicate the y-intercepts. If necessary, find a few additional points and graph the function.

Answer: The graph falls to the left and rises to the right, $x = -2, -\frac{1}{2}, 3$, $y = -6$. Crosses the x-axis at every zero since each zero has multiplicity 1.



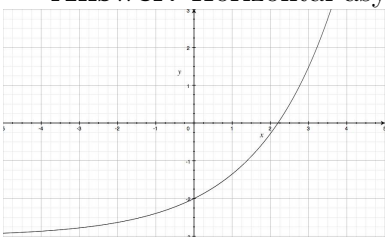
32. Graph the rational function $f(x) = \frac{3x}{x-2}$. Indicate all x-intercepts, y-intercepts, horizontal asymptote, vertical asymptote(s). If necessary, find a few additional points and graph the function.

Answer: $x = 0$, $y = 0$, vertical asymptote at $x = 2$, horizontal asymptote at $y = 3$.



33. Sketch the graph of the exponential function $f(x) = e^{\frac{1}{2}x} - 3$.

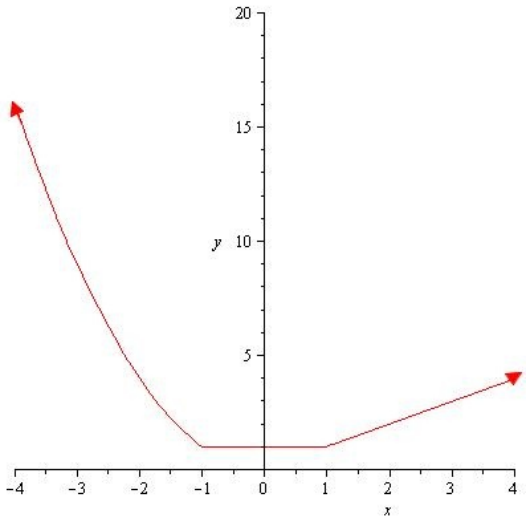
Answer: Horizontal asymptote located at $y = -3$



34. Graph the following Piecewise function and state the domain, range and intervals where the function is increasing, decreasing and constant.

$$G(x) = \begin{cases} x^2 & x < -1 \\ 1 & -1 \le x \le 1 \\ x & x > 1 \end{cases}$$

Answer: The domain of this function is the set of all Real Numbers, the range is the interval $[1, \infty)$. The interval of increasing is $(1, \infty)$, the interval of decreasing is $(-\infty, -1)$ and the interval where the function is constant is $(-1, 1)$. See the graph below.



E.10

35. Apply properties of Logarithms to simplify each expression.

a) $7^{2\log_7 5}$ b) $\log_5 \sqrt[3]{5}$

Answer: a) 25, b) $\frac{1}{3}$

36. Expand each expression by writing in terms of sum or difference of logarithms.

a) $\log_b(\frac{r^4}{s^2})$ b) $\ln[\frac{\sqrt{x+3}\sqrt[3]{x-4}}{(x+1)^4}]$

Answer: a) $4\log_b r - 2\log_b s$, b) $\frac{1}{2}\ln(x+3) + \frac{1}{3}\ln(x-4) - 4\ln(x+1)$

37. Write the expression as a single Logarithm.

$$2\log u - 3\log v - 2\log z$$

Answer: $\log(\frac{u^2}{v^3 z^2})$

38. Write the expression as a single Logarithm.

$$\ln\sqrt{x-1}+\ln\sqrt{x+1}-2\ln(x^2-1)$$

Answer: $\frac{3}{2}\ln(\frac{1}{x^2-1})$ or $\frac{-3}{2}\ln(x^2-1)$

E.11

39. Suppose that y is such that $\log_2 y = 17.67$. Evaluate $\log_2 y^{100}$
Answer: $\log_2 y^{100} = 1767$

40. Solve for all the values of x that satisfy the equation: $\log_3(x+5)+\log_3(x-1)=2$.
Answer: $-2+3\sqrt{2}$

41. Solve the equation by making an appropriate substitution; $2x^{\frac{2}{3}}+7x^{\frac{1}{3}}-15=0$.
Answer: $x=\frac{27}{8}, x=-125$

42. Solve the radical equation. Check the proposed solutions. $x-\sqrt{2x+5}=5$.
Answer: $x=10$

43. Find the rational zeros of f. List any irrational zero correct to two decimal places.
$f(x)=x^4+5x^3-3x^2-35x-28$.

Answer: Rational zeros: $x_1=-4, x_2=-1$, Irrational zeros: $x_3=2.65, x_4=-2.65$

44. Solve the exponential equation. Round your answer to four decimal places.

$$5^x = 4^{x-6}$$

Answer: $x=-37.2754$

45. Solve the radical equation. Check the proposed solutions. $\sqrt{x+2}+\sqrt{7x+2}=6$
Answer: $x=2$, note that $x=14$ does not satisfy the original equation.

46. Solve the exponential equation $(\frac{1}{2})^{4y}=16$.
Answer: $y=-1$.

E.12

47. Solve the inequality $x^2-x>12$.
Answer: $(-\infty,-3)\cup(4,\infty)$

48. Solve the inequality $x^3-x^2\leq 6x$.
Answer: $(-\infty,-2]\cup[0,3]$

49. Solve the inequality $\dfrac{x+2}{x-1} \leq 0$.

Answer: $[-2, 1)$

50. Solve the inequality and write your answer in interval notation; $\dfrac{5x+1}{x} < 1$

Answer: $(-1/4, 0)$

51. A Web-based embroidery company makes monogrammed napkins. The profit associated with producing x orders of napkins is governed by the equation

$$P(x) = -x^2 + 130x - 3000$$

Determine how many orders the company should accept to make a profit.

Answer: 31 to 99 orders will yield a profit. (i.e. $(30, 100)$).

E.13

52. State the order of the matrix

$$\begin{bmatrix} 3 & 5 \\ 2 & 6 \\ -1 & -4 \end{bmatrix}$$

Answer: 3 x 2, (i.e 3 rows by 2 columns)

53. Solve for the indicated variables.

$$\begin{bmatrix} 3 & 4 \\ 0 & 12 \end{bmatrix} = \begin{bmatrix} x-y & 4 \\ 0 & 2y+x \end{bmatrix}$$

Answer: $x = 6$, $y = 3$

54. Let $A = \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 2 & 5 \end{bmatrix}$. Find $2A - 3B$ and BA.

Answer: $2A - 3B = \begin{bmatrix} -4 & 1 \\ -6 & -9 \end{bmatrix}$, and $BA = \begin{bmatrix} 2 & -5 \\ 2 & 13 \end{bmatrix}$.

55. The IRS allows and individual to deduct business expenses in the following way: \$0.45 per mile driven, 50% of entertainment cost, and 100% of actual expenses. Represent these deductions as a row matrix A. In 2006, Joe had the following business expenses: $\$2,700$ in entertainment, $\$15,200$ actual expenses, and he drove 7523 miles. Represent Joe's expenses as a column matrix B. Multiply these matrices (AB) to find the total amount of business expenses Joe can claim on his 2006 tax form.

Answer: $A = \begin{bmatrix} 0.45 & 0.5 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 7,523 \\ 2,700 \\ 15,200 \end{bmatrix}$, $AB = \$19,935.35$.

56. Find the inverse (if it exists) of the matrix $A = \begin{bmatrix} -1 & -3 \\ -4 & 3 \end{bmatrix}$.

Answer: $A^{-1} = \begin{bmatrix} -\frac{1}{5} & -\frac{1}{5} \\ -\frac{4}{15} & \frac{1}{15} \end{bmatrix}$

E.14

57. Find the fifth term in the expansion of $(s - 3y)^7$.
Answer: The fifth term is $2835s^3y^4$.

58. Apply the binomial theorem to expand $(2z - 3y)^4$.
Answer: $(2z - 3y)^4 = 16z^4 - 96z^3y + 216z^2y^2 - 216zy^3 + 81y^4$

E.15

59. The 54th and 4th term of an arithmetic sequence are -61 and 64, respectively. Find the 23rd term in the sequence.
Answer: the 23rd term is $\frac{33}{2}$

60. Find the sum of the first 11 terms of the Arithmetic sequence $9, 17, 25, 33, ...$
Answer: 539.

61. Find the first four terms and the 10th term of the sequence given by $s(n) = \frac{n(n+2)}{4}$.
Answer: $\frac{3}{4}, 2, \frac{15}{4}, 6$; 30.

62. Give the first four terms of the geometric sequence for which $a_1 = -5$ and $r = 4$.
Answer: $-5, -20, -80, -320$

63. Find the next three terms of the geometric sequence $27, -9, 3, -1, ...$
Answer: $\frac{1}{3}, -\frac{1}{9}, \frac{1}{27}$.

64. For the sequence $5, -1, \frac{1}{5}, -\frac{1}{25}, ...$. write a formula for the general term a_n.
Answer: $a_n = 5(-\frac{1}{5})^{n-1}$ or $a_n = -25(-\frac{1}{5})^n$.

65. Find the infinite sum $0.5 + 0.05 + 0.005 + ...$.
Answer: $\frac{5}{9}$.

66. Find the sum of the first seven terms of the sequence $\frac{108}{64}, \frac{36}{16}, \frac{12}{4}, ...$
Answer: $\frac{14{,}197}{432} \approx 32.863$.

E.16

67. Use **Cramer's Rule** to solve for x and y.

$$\begin{aligned} 3x - 4y &= 1 \\ 4x - 2y &= 8 \end{aligned}$$

Answer: $x = 3$ and $y = 2$

68. Solve the system of equations by substitution or addition method:

$$\begin{aligned} x + y + z &= 0 \\ 2x + z &= -1 \\ x - y - z &= 2 \end{aligned}$$

Answer: $x = 1, y = 2, z = -3$

69. Solve the system by the method of your choice:

$$\begin{aligned} 2u + 5v &= 7 \\ 3u - v &= 5 \end{aligned}$$

Answer: $(\frac{32}{17}, \frac{11}{17})$

70. A Honda Accord gets approximately 26 mpg on the highway and 19 mpg in the city. You drove 349.5 miles on a full 16 gallon tank. Assuming you drove on both highway and city, approximately how many miles did you drive in the city and how many on the highway?

Answer: 169 miles on the highway, 180.5 miles in the city .

71. Find the determinant

$$\begin{vmatrix} 6 & 4 & 0 \\ -3 & -5 & 3 \\ 1 & 2 & 0 \end{vmatrix}$$

Answer: -24

E. 17

72. Prove by mathematical induction that, for all positive integers n,

$$1 + 2 + 3 + 4 + ... + n = \frac{n(n+1)}{2}.$$

Solution:

STEP 1. First we show that the formula above holds true when $n = 1$; i.e. $1 = \frac{1(1+1)}{2} = 1$. $\surd$

STEP 2. Now we assume that the formula above holds true for any $k < n$, that is

$$1 + 2 + 3 + 4 + ... + k = \frac{k(k+1)}{2}.\surd$$

STEP 3. If we can show that the formula above holds for $k + 1$ term, we are done. So, adding $k + 1$ to both sides in step 2 we get,

$$1 + 2 + 3 + 4 + ... + k + (\mathbf{k+1}) = (\mathbf{k+1}) + \frac{k(k+1)}{2}$$

$$1 + 2 + 3 + 4 + ... + k + (\mathbf{k+1}) = \frac{2}{2}(\mathbf{k+1}) + \frac{k(k+1)}{2}$$

$$1 + 2 + 3 + 4 + ... + k + (\mathbf{k+1}) = \frac{2(\mathbf{k+1}) + k(k+1)}{2}$$

$$1 + 2 + 3 + 4 + ... + k + (\mathbf{k+1}) = \frac{(k+1)(k+2)}{2}$$

note that the right hand side is the value $\frac{n(n+1)}{2}$ when $(k + 1)$ is substituted for n.
Hence if the formula is true for k, we have proved it is also true for $k + 1$, but since k is any positive integer the formula is true for ALL positive integers n.
...Please see your instructor or tutor for a more detailed reasoning.

Extras:

73. Find the center and radius given the following equation of a circle:

$$x^2 + y^2 + 6x + 2y + 6 = 0$$

Answer: Center: $(-3, -1)$ radius: $r = 2$

74. Use the Remainder Theorem to find $P(-5)$ if $P(x) = x^6 + 6x^5 + 5x^3 - 6x^2 - 24$
Answer: $P(-5) = -3924$

75. Find the sum $\sum_{n=0}^{3} \frac{2}{n!}$
Answer: $\frac{16}{3}$

*__NOTE:__ The problems in this document are a selection of problems from many sources, including College Algebra textbooks, Mymathlab, Wileyplus and Eastfield College math faculty. This document should give you an idea of the topics you need know after completing College Algebra, this document does not intend to give information on any type of examination, including the final exam.